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AXIAL COMPRESSION OF AN INHOMOGENEOUS CONE

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UDC 539.376

A concentrated axial force is applied to the apex of an infinite cone. The material is assumed to be incompressible, inhomogeneous, and conforming to the power law of reinforcement. The compression of this cone is investigated. The corresponding homogeneous problem was studied in [1, 2].

Differential equations of equilibrium for an axisymmetric deformation in spherical coordinates in the usual notations appear as

$$\frac{\partial\sigma_r}{\partial r}+\frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta}+\frac{1}{r}(2\sigma_r-\sigma_\theta-\sigma_\varphi+\tau_{r\theta}\operatorname{ctg}\theta)=0,$$
$$\frac{\partial\tau_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_\theta}{\partial\theta}+\frac{1}{r}[(\sigma_\theta-\sigma_\varphi)\operatorname{ctg}\theta+3\tau_{r\theta}]=0,$$
$$\frac{\partial\tau_{r\varphi}}{\partial r}+\frac{1}{r}\frac{\partial\tau_{\theta\varphi}}{\partial\theta}+\frac{1}{r}(3\tau_{r\varphi}+2\tau_{\theta\varphi}\operatorname{ctg}\theta)=0. \quad (1)$$

The reinforcement law for this material is

$$\sigma_0=k\omega(\theta)\varepsilon_0^m,\quad 0<m<1, \quad (2)$$

where k is a constant; ω(θ) is a given function determined experimentally;

$$\sigma_0=\frac{1}{\sqrt{6}}\sqrt{(\sigma_r-\sigma_\theta)^2+(\sigma_\theta-\sigma_\varphi)^2+(\sigma_\varphi-\sigma_r)^2+6\left(\tau_{r\theta}^2+\tau_{\theta\varphi}^2+\tau_{r\varphi}^2\right)},$$
$$\varepsilon_0=\sqrt{\frac{2}{3}}\sqrt{(\varepsilon_r-\varepsilon_\theta)^2+(\varepsilon_\theta-\varepsilon_\varphi)^2+(\varepsilon_\varphi-\varepsilon_r)^2+6\left(\gamma_{r\theta}^2+\gamma_{\theta\varphi}^2+\gamma_{r\varphi}^2\right)}$$

Erevan. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 115-118, November-December, 1990. Original article submitted November 9, 1988; revision submitted May 4, 1989.

are the intensities of tangential stresses and shear strains, respectively.

The relations between the components of strains, displacements, and stresses in case of axisymmetric strain are defined by the expressions

$$\varepsilon_r = \frac{\partial u}{\partial r},\ \varepsilon_\theta = \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta},\ \varepsilon_\varphi = \frac{u}{r} + \frac{v}{r}\operatorname{ctg}\theta,\ 2\gamma_{r\theta} = \frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r}\frac{\partial u}{\partial \theta}, \tag{3}$$
$$2\gamma_{\theta\varphi} = \frac{1}{r}\frac{\partial w}{\partial \theta} - \frac{w}{r}\operatorname{ctg}\theta,\ 2\gamma_{r\varphi} = \frac{\partial w}{\partial r} - \frac{w}{r},$$
$$\varepsilon_{ij} = \frac{1}{2}k^{-1/m}\omega^{-1/m}(\theta)\sigma_0^{1/m-1}(\sigma_{ij} - \delta_{ij}\sigma)$$

[δ_{ij} is the Kronecker symbol, $\sigma = (1/3)(\sigma_r + \sigma_\theta + \sigma_\varphi)$ is the average stress]. The displacement components that satisfy the incompressibility condition can be expressed as

$$u(r, \theta) = Ar^{1-\lambda}[\psi'(\theta) + \psi(\theta)\operatorname{ctg}\theta], \tag{4}$$
$$v(r, \theta) = A(\lambda - 3)r^{1-\lambda}\psi(\theta),\ w(r, \theta) = 0,$$

where A and λ are unknown constant parameters and $\Psi(\theta)$ is an arbitrary unknown function to be defined.

Proceeding from relations (3) and (4), we have for the stress components the expressions

$$\sigma_r = \sigma_\theta - 2A^m kr^{-\lambda m}[(2\lambda - 3)\psi'(\theta) + \lambda\psi(\theta)\operatorname{ctg}\theta]\omega(\theta)\chi(\theta), \tag{5}$$
$$\sigma_\varphi = \sigma_\theta - 2A^m k(\lambda - 3)r^{-\lambda m}[\psi'(\theta) - \psi(\theta)\operatorname{ctg}\theta]\omega(\theta)\chi(\theta),$$
$$\tau_{r\theta} = A^m kr^{-\lambda m}[\psi''(\theta) + (\psi\operatorname{ctg}\theta)' + \lambda(3 - \lambda)\psi(\theta)]\omega(\theta)\chi(\theta),$$
$$\tau_{r\varphi} = 0,\ \tau_{\theta\varphi} = 0,$$

where

$$\chi(\theta) = \{[\psi''(\theta) + (\psi\operatorname{ctg}\theta)' + \lambda(3 - \lambda)\psi(\theta)]^2 + \tag{6}$$
$$+ 4(\lambda^2 - 3\lambda + 3)(\psi'^2(\theta) + \psi^2(\theta)\operatorname{ctg}^2\theta) + 4(\lambda^2 - 3)\psi(\theta)\psi'(\theta)\operatorname{ctg}\theta\}^{(m-1)/2}.$$

Substituting stress expressions (5) into the first equilibrium equation in (1), we obtain after integration

$$\sigma_\theta = \frac{A^m kr^{-\lambda m}}{\lambda m}\{2(\lambda m - 2)[(2\lambda - 3)\psi'(\theta) + \lambda\psi(\theta)\operatorname{ctg}\theta]\omega(\theta)\chi(\theta) + \tag{7}$$
$$+ [(\psi''(\theta) + (\psi\operatorname{ctg}\theta)' + \lambda(3 - \lambda)\psi(\theta))\omega(\theta)\chi(\theta)]' +$$
$$+ [\psi''(\theta) + (\psi\operatorname{ctg}\theta)' + \lambda(3 - \lambda)\psi(\theta)]\omega(\theta)\chi(\theta)\operatorname{ctg}\theta + 2(\lambda - 3)[\psi'(\theta) - \psi(\theta)\operatorname{ctg}\theta]\omega(\theta)\chi(\theta) + c$$

(c is the integration constant).

The boundary conditions of this problem are the following:

$$\sigma_\theta = \tau_{r\theta} = 0 \text{ at } \theta = \alpha;\ \tau_{r\theta} = 0,\ v = 0 \text{ at } \theta = 0. \tag{8}$$

They are combined with the condition of static equilibrium of an imaginary conical body bounded by an arbitrary spherical surface of radius r with its center at the apex:

$$p + 2\pi\int_0^\alpha(\sigma_r\cos\theta - \tau_{r\theta}\sin\theta)r^2\sin\theta\,d\theta = 0. \tag{9}$$

By virtue of (9), $\lambda m = 2$ and $c = 0$. Satisfying this second equilibrium equation in (1) in the search for the unknown function $\Psi(\theta)$, we obtain a fourth-order nonlinear ordinary differential equation:

$$[(\psi'' + (\psi\operatorname{ctg}\theta)' + \lambda(3 - \lambda)\psi)\omega\chi]'' + [(\psi'' + (\psi\operatorname{ctg}\theta)' + \tag{10}$$
$$+ \lambda(3 - \lambda)\psi)\omega\chi\operatorname{ctg}\theta]' + 2[\psi'' + (\psi\operatorname{ctg}\theta)' + \lambda(3 - \lambda)\psi]\omega\chi +$$
$$+ 2(\lambda - 3)[(\psi' - \psi\operatorname{ctg}\theta)\omega\chi]' + 4(\lambda - 3)(\psi' - \psi\operatorname{ctg}\theta)\omega\chi\operatorname{ctg}\theta = 0.$$

Equation (10) is solved numerically. It is convenient to reduce this equation to a set of four first-order differential equations:

$$\psi' = s - \psi\operatorname{ctg}\theta,\ s' = \tau - \lambda(3 - \lambda)\psi, \tag{11}$$
$$\tau' = \frac{F - T}{\omega\chi + (m - 1)\tau^2\chi^{(m-3)/(m-1)}},\ F' = -2\tau\omega\chi - 4(\lambda - 3)(s - 2\psi\operatorname{ctg}\theta)\omega\chi\operatorname{ctg}\theta,$$

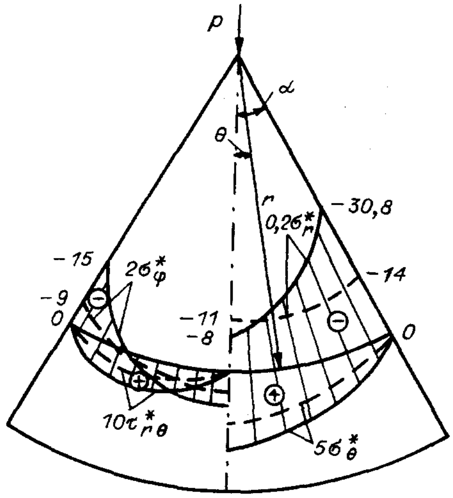

Fig. 1

where $\chi = \{\tau^2 + 4(\lambda^2 - 3\lambda + 3)s^2 - 4(\lambda - 3)^2(s - \Psi \operatorname{ctg}\theta)\psi \operatorname{ctg}\theta)^{(m-1)/2}$;

$$T = \omega'\tau\chi + \omega\tau\chi \operatorname{ctg}\theta + 2(\lambda - 3)(s - 2\psi \operatorname{ctg}\theta)\omega\chi + \\ + ((m-1)/2)\chi^{(m-3)/(m-1)}\{8(\lambda^2 - 3\lambda + 3)s[\tau - \lambda(3-\lambda)\psi] \\ - 4(\lambda - 3)^2[(\tau - \lambda(3-\lambda)\psi - (s - \psi \operatorname{ctg}\theta)\operatorname{ctg}\theta + \psi/\sin^2\theta)\psi \operatorname{ctg}\theta + \\ + (s - \psi \operatorname{ctg}\theta)((s - \psi\operatorname{ctg}\theta)\operatorname{ctg}\theta - \psi/\sin^2\theta)]\}.$$

The boundary conditions of problem (8) for system (11) appear as

$$\psi = 0, \tau = 0 \text{ at } \theta = 0; F = 0, \tau = 0 \text{ at } \theta = \alpha. \tag{12}$$

The final formulas for stresses and displacements (4)-(7) can be written as

$$\sigma_r = (A^m k/2r^2)\{F(\theta) - 4[(2\lambda - 3)s(\theta) + (3-\lambda)\psi(\theta)\operatorname{ctg}\theta]\omega(\theta)\chi(\theta)\},$$
$$\sigma_\varphi = (A^m k/2r^2)[F(\theta) - 4(\lambda - 3)(s(\theta) - 2\psi(\theta)\operatorname{ctg}\theta)\omega(\theta)\chi(\theta)],$$
$$\sigma_\theta = (A^m k/2r^2)F(\theta),\ \tau_{r\theta} = (A^m k/r^2)\tau(\theta)\omega(\theta)\chi(\theta),$$
$$\tau_{r\varphi} = 0,\ \tau_{\theta\varphi} = 0,\ u = Ar^{1-\lambda}s(\theta),\ v = A(\lambda - 3)r^{1-\lambda}\psi(\theta),\ w = 0.$$

The unknown constant A is determined from the condition of static equilibrium (9).

On the basis of numeric solution of the boundary-value problem (11) and (12), we plotted on an ES-1022 computer, by the zero-in method of [3], the curves of relative stresses $\sigma_{ij}^* = (2r^2/A^m k)\cdot\sigma_{ij}$ at $\alpha = \pi/6$ and the inhomogeneity $\omega(\theta) = \exp(\mu\theta)$ (see Fig. 1; solid lines refer to an inhomogeneous material; dashed lines refer to a homogeneous material). For this numeric example we assumed $\mu = 2.098$.

The inhomogeneity of the mechanical properties of materials can be due to a variety of factors: the effect of the elementary particle flux, temperature gradients, nonuniform reinforcement of the material, surface finishing, etc.

A comparison of these data with the results of a homogeneous material indicates that the inhomogeneity has a significant effect on the stressed–strained state.

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